

yet calculated the stress fields and correlated them with experiments or with fracture formulae.

An interesting property of iron and carbon steels is their propensity to fracture under explosive or projectile impact so as to produce a very smooth surface. This fracture is called the «smooth spall» and has been described by RINEHART and PEARSON [14], by ERKMAN [17], by LETHABY and SKIDMORE [18], and by TUPPER [19]. In the last three works the smooth spall is attributed to the interaction of rarefaction shocks associated with the  $\alpha$ - $\epsilon$  phase transition in iron.

### 9. - Shock waves on a one-dimensional lattice of mass points.

The mass points are connected by nonlinear springs, as in Fig. 31, and by dashpots. The forces exerted by the former on the  $N$ -th point mass are denoted

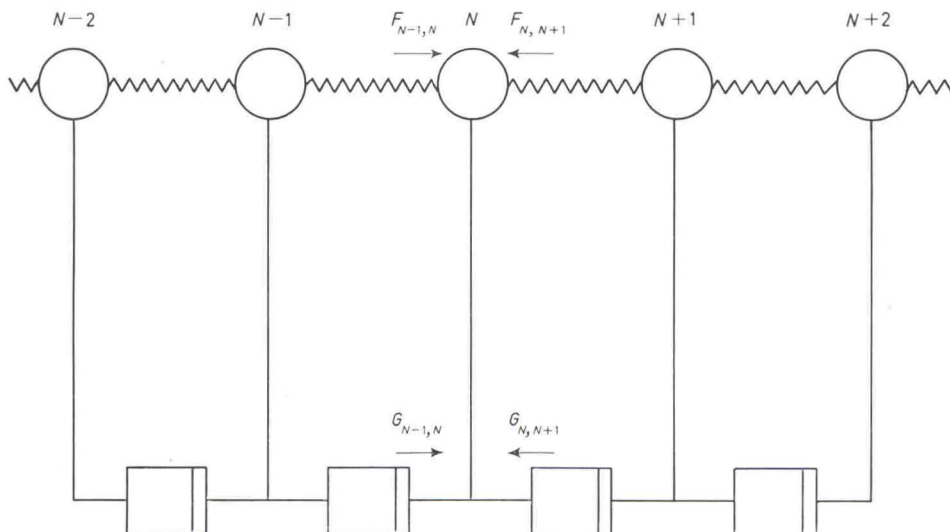


Fig. 31. - Dissipating lattice model.  $d^2X_N/dT^2 = F_{N-1,N} - F_{N,N+1} + G_{N-1,N} - G_{N,N+1}$ .

by  $F_{N-1,N}$  and  $F_{N,N+1}$ ; those associated with the dashpots are  $G_{N-1,N}$  and  $G_{N,N+1}$ . The equation of motion of the  $N$ -th particle is

$$(103) \quad d^2X_N/dT^2 = F_{N-1,N} - F_{N,N+1} + G_{N-1,N} - G_{N,N+1},$$

where  $X_N$  is its position at time  $T$ ;  $X$  and  $T$  are reduced values corresponding to unit mass and unit equilibrium separation between mass points. The lattice

is supposed to extend from  $N=1$  to  $\infty$  and we imagine the first mass point to be given a velocity  $u_1$  at  $T=0$ . The entire lattice is assumed quiescent before that. For a parabolic force law,

$$(104) \quad F_{N,N+1} = -(S_{N+1} - S_N) + \alpha(S_{N+1} - S_N)^2, \quad S_N = X_N - 1, \quad G_{N,N+1} = 0,$$

the acceleration of the  $N$ -th particle is shown in Fig. 32 [20]. The broken line shown there is the result of a numerical integration, the dashed line is

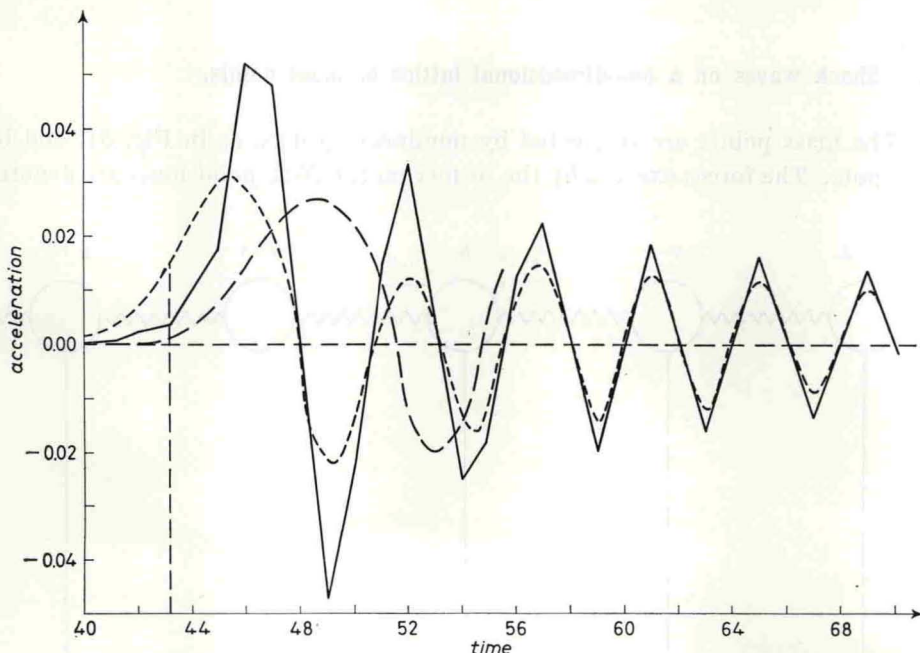


Fig. 32. - Acceleration of a mass point in a nonlinear lattice. (Ref. [20]).

an analytic approximation obtained by averaging the nonlinear force term over time, and the long-dashed line is the exact solution when the coefficient  $\alpha=0$  in eq. (104). If solutions are compared for successively larger values of  $N$ , it is found that the initial acceleration increases monotonically with  $N$  and that the rate of decay of the oscillation decreases monotonically. This suggests that the solution may be approaching a permanent regime:

$$(105) \quad S_N(T) = S(T - N\theta),$$

where  $\theta$  is a constant equal to the reciprocal wave speed. When eq. (105) is